Group-invariant operators on the Bergman space over the unit ball: analysis via the Fourier transforms of the reproducing kernel

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Toeplitz Operators, Wiener-Hopf Method, and Applications

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Outline

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$\mathbb{D} := \{z \in \mathbb{C} \colon \ |z| < 1\}, \qquad \mathbb{T} := \{z \in \mathbb{T} \colon \ |z| = 1\}.$

$\mathcal{A}^2(\mathbb{D}) :=$ the Bergman space on $\mathbb{D}$.

$\text{Möb}(\mathbb{D}) :=$ the Möbius transformations preserving $\mathbb{D}$.

$P \colon L^2(\mathbb{D}) \to L^2(\mathbb{D})$ such that $P(L^2(\mathbb{D})) = \mathcal{A}^2(\mathbb{D})$, the Bergman projection.

Given $a \in L^\infty(\mathbb{D})$, Toeplitz operator $T_a \colon \mathcal{A}^2(\mathbb{D}) \to \mathcal{A}^2(\mathbb{D})$,

$$T_a f := P(af), \qquad \text{i.e.,} \qquad (T_a f)(z) = \int_{\mathbb{D}} f(w) a(w) \overline{K_z^{\mathcal{A}^2(\mathbb{D})}(w)} \, \mathrm{d}\mu(w).$$

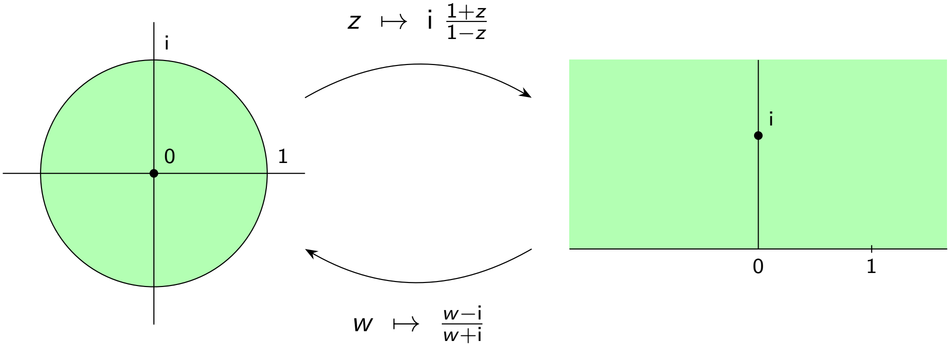
Cayley transform: a biholomorfism between Π and $\mathbb{D}$

unit disk

$$\mathbb{D} = \{z \in \mathbb{C} \colon |z| < 1\}$$

upper halfplane

$$\Pi = \{w \in \mathbb{C} \colon \operatorname{Im}(w) > 0\}$$



Discovering of non-trivial commutative algebras of Toeplitz operators

Vasilevski (1999):
On Bergman-Toeplitz operators with commutative symbol algebras.

Grudsky, Karapetyants, Vasilevski (2004):
Dynamics of properties of Toeplitz oper. on the upper half-plane: Hyperbolic case.

Grudsky, Quiroga-Barranco, Vasilevski (2006):
Commutative C*-algebras of Toeplitz operators and quantization on the unit disk.

Vasilevski (2008):
Commutative Algebras of Toeplitz Operators on the Bergman Space.

Three cases on the unit disk

Three types of maximal abelian subgroups of Möb($\mathbb{D}$):

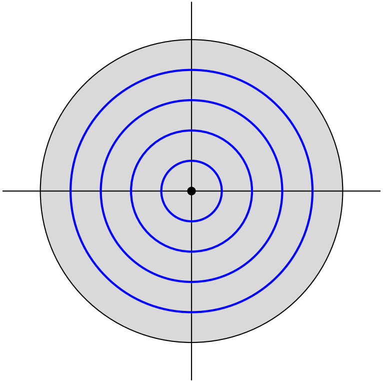
- elliptic,
- parabolic,
- hyperbolic.

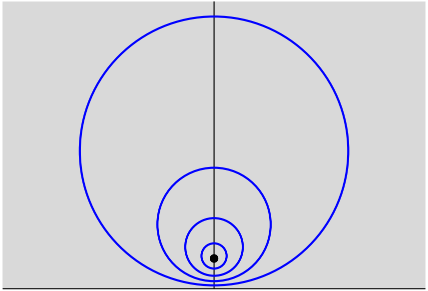
For each of these subgroups G,
they considered the generating symbols that are constant on the orbits:

$$L_G^\infty(\mathbb{D}) := \left\{ a \in L^\infty(\mathbb{D}) : \quad \forall \varphi \in G \quad a \circ \varphi \xlongequal{\text{a.e.}} a \right\}.$$

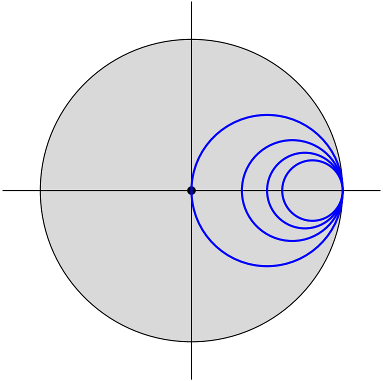
Elliptic subgroup (radial generating symbols on $\mathbb{D}$)

$$\varphi(z) = \tau z, \quad \tau \in \mathbb{T}$$

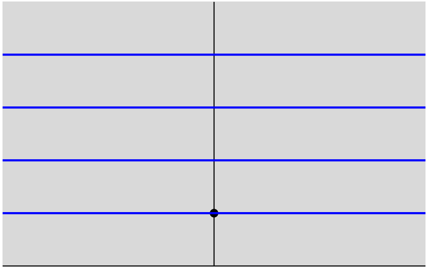




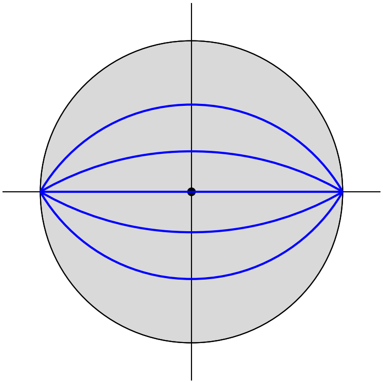
Parabolic subgroup (vertical generating symbols on Π)



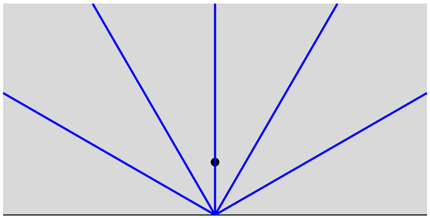
$$\varphi(w) = w + h, \quad h \in \mathbb{R}$$



Hyperbolic subgroup (angular generating symbols on Π)



$$\varphi(w) = d\,w, \quad d > 0$$



Commutative algebras of Toeplitz operators

For each of these subgroups G,

they constructed a topological and measure space Ω and an isometric isomorphism

$$R\colon \mathcal{A}^2(\mathbb{D}) \to L^2(\Omega)$$

that simultaneously diagonalizes all Toeplitz operators T_a with a in $L_G^\infty(\mathbb{D})$:

$$RT_aR^* = M_{\gamma_a}, \qquad \gamma_a \in C_b(\Omega).$$

As a consequence, $\{T_a\colon\ a \in L_G^\infty(\mathbb{D})\}$ generates a commutative C*-algebra.

Multidimensional situation

Unit ball $\mathbb{B}_n$ in $\mathbb{C}^n$:
$$\{z \in \mathbb{C}^n : \ |z| < 1\}$$

$\longleftarrow$ Cayley transform $\longrightarrow$

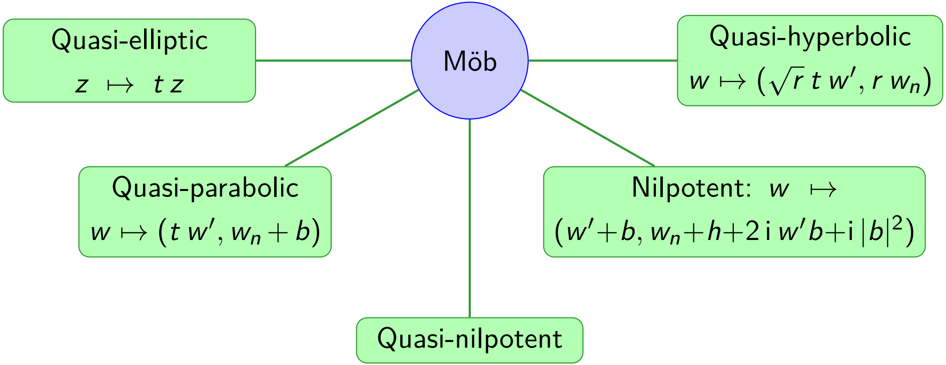
Siegel domain D_n in $\mathbb{C}^n$:
$$\{w \in \mathbb{C}^n : \ \operatorname{Im}(w_n) > |w'|^2\}$$
$$w = (\underbrace{w_1, \ldots, w_{n-1}}_{w'}, w_n)$$

Quiroga-Barranco, Vasilevski (2007):

Commutative C*-algebras of Toeplitz operators on the unit ball, I.

Bargmann-type transforms and spectral representations of Toeplitz operators.

Maximal commutative subgroups of Möb($\mathbb{B}_n$) or Möb(D_n)



A scheme used by Vasilevski and his colleagues

$\mathcal{A}^2\colon \; \frac{\partial f}{\partial \overline{z}} = 0,$
$\mathcal{A}^2 < L^2,$
$g \bullet z$

$\xrightarrow{U_1}$ (weighted change of var.)

new dif. eq.
new L^2

$\downarrow$ $U_2 = F \otimes I$ | $U_3 = ?$

isometr. isomorph.
$R\colon \mathcal{A}^2 \to L^2(\Omega)$
$R = U_3 U_2 U_1$

$\longleftarrow$ direct computation

$R T_a R^* = M_{\gamma_a},$
$\gamma_a \in C_b(\Omega)$

Our approach is slightly different

$\mathcal{A}^2 = \text{RKHS}$
$\rho\colon G \to \mathcal{U}(\mathcal{A}^2)$

U (weighted change of var.) →

RKHS H
kernel K
$U\rho = \rho_H U$

↓ $F \otimes I$

$L = (F \otimes I)K$
$L_{\xi,y}(v) = \overline{q_\xi(y)}q_\xi(v)$

← gratis (automatically)

$R\colon \mathcal{A}^2 \to L^2(\Omega),$
$R\mathcal{C}(\rho)R^* \cong L^\infty(\Omega)$

Outline

Main tool: a scheme for tube-type domains $G \times Y$

Herrera-Yañez, Maximenko, Ramos-Vazquez (2022):
Translation-invariant operators in reproducing kernel Hilbert spaces.

- $X = G \times Y$,
- G is an abelian locally compact group, metrizable, and σ-compact,
- ν is a Haar measure on G,
- (Y, λ) is a σ-finite measure space,
- $L^2(G \times Y)$ is separable.

Horizontal translations acting in $L^2(G \times Y)$

G acts on $G \times Y$ by

$$\tau_{G\times Y}(g)\colon \quad (x,y) \mapsto (g+x,y),$$

For each g in G, define $\rho_{G\times Y}(g)\colon L^2(G\times Y) \to L^2(G\times Y)$,

$$(\rho_{G\times Y}(g)f)(x,y) := f(x-g,y),$$

$\rho_{G\times Y}$ is a unitary representation of G in $L^2(G\times Y)$.

Our assumptions

- H is a closed subspace of $L^2(G \times Y)$.
- H is an RKHS; we denote the RK by $(K_{x,y})_{x\in G,\ y\in Y}$.
- H is invariant under $\rho_{G\times Y}$.
  Equivalently,
  $$K_{x,y}(u,v) = K_{0,y}(u-x,v).$$
- $\forall y \in Y \quad \sup_{v\in Y} \int_G |K_{(0,y)}(u,v)|\,\mathrm{d}\nu(u) < +\infty.$

The projection over H is a convolution wrt the first coordinate!

$$
\begin{aligned}
(Pf)(x,y) &= \langle f, K_{x,y}\rangle \\
&= \int_G \int_Y f(u,v)\overline{K_{x,y}(u,v)}\,\mathrm{d}\nu(u)\,\mathrm{d}\lambda(v) \\
&= \int_G \int_Y f(u,v) K_{0,v}(x-u,y)\,\mathrm{d}\nu(u)\,\mathrm{d}\lambda(v).
\end{aligned}
$$

So, it is natural to apply $F \otimes I$.

$$
L_{\xi,y}(v) := \int_G \overline{\xi(u)} K_{0,y}(u,v)\,\mathrm{d}\nu(u) \qquad \xi \in \widehat{G}.
$$

Decomposition of $\widehat{H} := (F \otimes I)H$

For each ξ in $\widehat{G}$,

$$\widehat{H}_\xi \;:=\; \operatorname{clos}\left(\operatorname{span}\{L_{\xi,y}\colon\; y \in Y\}\right) \;\le\; L^2(Y).$$

Then $\widehat{H}_\xi$ is a RKHS with kernel $L_{\xi,\bullet}$.

$$\Omega := \left\{\xi \in \widehat{G}\colon\;\; \dim(\widehat{H}_\xi) > 0\right\}.$$

$$\widehat{H} := (F \otimes I)H = \int_\Omega^\oplus \widehat{H}_\xi \,\mathrm{d}\widehat{\nu}(\xi).$$

Idea of the proof: convolution theorem + Fubini + Moore–Aronszajn theorem.

Horizontal translations acting H

For each g in G, let $\rho_H(g)\colon H \to H$ be the compression of $\rho_{G\times Y}(g)$:

$$(\rho_H(g)f)(u,v) := f(u-g,v) \qquad (f \in H).$$

Then ρ_H is a unitary representation of G in H.

Consider the centralizer of ρ:

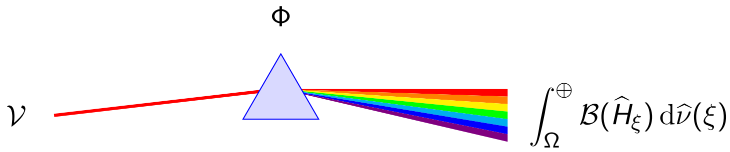
$$\mathcal{V} := \mathcal{C}(\rho_H) := \Big\{S \in \mathcal{B}(H)\colon \quad \forall g \in G \quad \rho(g)S = S\rho(g)\Big\}.$$

Decomposition of $\mathcal{C}(\rho_H)$

Let $\Phi\colon H \to \widehat{H}$ be the compression of $F \otimes I$.

Theorem

$$\Phi\,\mathcal{V}\,\Phi^* = \int_\Omega^\oplus \mathcal{B}(\widehat{H}_\xi)\,\mathrm{d}\widehat{\nu}(\xi).$$



Constructive criterion for the commutativity of $\mathcal{V}$

Theorem

The following conditions are equivalent.

(a) $\mathcal{V}$ is commutative.

(b) $d_\xi := \dim(\widehat{H}_\xi) = 1$ for every ξ in Ω.

(c) $|L_{\xi,y}(v)|^2 = L_{\xi,y}(y)L_{\xi,v}(v)$ for every ξ in Ω and every y, v in Y.

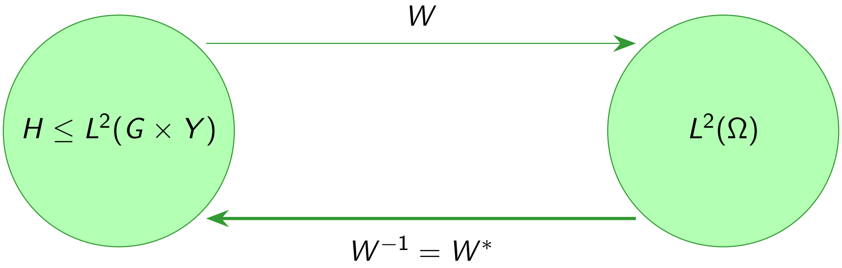
(d) There exists a family $(q_\xi)_{\xi\in\Omega}$ in $L^2(Y)$ such that the function $(\xi, v) \mapsto q_\xi(v)$ is measurable, $\widehat{H}_\xi = \mathbb{C}q_\xi$, $\|q_\xi\| = 1$, and

$$L_{\xi,y}(v) = \overline{q_\xi(y)}q_\xi(v) \qquad (\xi \in \Omega,\ y, v \in Y).$$

Isometric isomorphism $W\colon H \to L^2(\Omega)$ in the commutative case

Suppose that $\dim(\widehat{H}_\xi) = 1$, i.e.,
there is a family $(q_\xi)_{\xi\in\Omega}$ such that $\widehat{H}_\xi = \mathbb{C}q_\xi$ y $\|q_\xi\| = 1$.

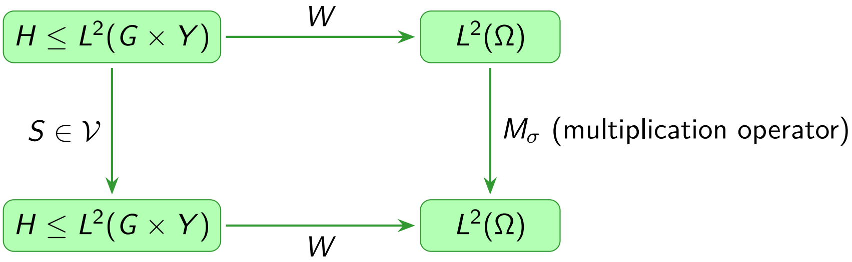
$$(Wf)(\xi) := \langle (\Phi f)(\xi,\cdot), q_\xi \rangle_{L^2(Y)}.$$



Diagonalization of translation-invariant operators in the case $d_\xi = 1$

Theorem

Suppose that $\dim(\widehat{H}_\xi) = 1$ for every ξ in Ω. Then $\mathcal{V} \cong L^\infty(\Omega)$.



Diagonalization of Toeplitz operators with translation-invariant generating symbols

Corollary

Let $a \in L^\infty(Y)$,

$$b(x, y) := a(y).$$

Then $T_b \in \mathcal{V}$, $WT_bW^* = M_{\gamma_a}$,

$$\gamma_a(\xi) := \int_Y a(v)\,|q_\xi(v)|^2\,\mathrm{d}\lambda(v) = \int_Y a(v) L_{\xi,v}(v)\,\mathrm{d}\lambda(v).$$

Outline

1. Introduction
2. Scheme for domains $G \times Y$
3. Quasi-elliptic subgroup
4. Quasi-parabolic subgroup

$\mathcal{A}^2(\mathbb{B}_n, \mu_{n,\alpha})$, the analytic Bergman space

$$\mathbb{B}_n := \{z \in \mathbb{C}^n \colon\ |z| < 1\}.$$

$$\mathrm{d}\mu_{n,\alpha}(z) = c_{n,\alpha}\,(1 - |z|^2)^\alpha\,\mathrm{d}\mu_{2n}(z), \qquad c_{n,\alpha} = \frac{\Gamma(n+\alpha+1)}{\pi^n\,\Gamma(\alpha+1)}.$$

$\mathcal{A}^2 = \mathcal{A}^2(\mathbb{B}_n, \mu_{n,\alpha}) :=$ holomorphic functions belonging to $L^2(\mathbb{B}_n, \mu_{n,\alpha})$.

Orthonormal basis: $\qquad b_j(z) = \sqrt{\dfrac{\Gamma(n+|j|+\alpha+1)}{j!\,\Gamma(n+\alpha+1)}}\,z^j, \qquad j \in \mathbb{N}_0^n.$

Reproducing kernel of $\mathcal{A}^2$: $\qquad K_z^{\mathcal{A}^2}(w) = \dfrac{1}{(1 - \langle w, z\rangle)^{n+1+\alpha}}.$

Group $\mathbb{R}_{2\pi}^n$ and its dual group

$G := \mathbb{R}_{2\pi}^n \cong \mathbb{T}^n$, where $\mathbb{R}_{2\pi} := \mathbb{R}/(2\pi\mathbb{Z})$.

$\nu :=$ the normalized Haar measure on G.

$$\int_G f \, \mathrm{d}\nu = \frac{1}{(2\pi)^n} \int_{[0,2\pi)^n} f(g) \, \mathrm{d}\mu_n(g).$$

$\widehat{G} = \mathbb{Z}^n$ with the counting measure $\widehat{\nu}$.

Rotations acting in $\mathcal{A}^2(\mathbb{B}_n)$

$G = \mathbb{R}^n_{2\pi} \cong \mathbb{T}^n$.

Möbius transforms associated to G:

$$\tau_{\mathrm{rot}}(g)(z) := \left(\mathrm{e}^{\mathrm{i}\, g_1} z_1, \ldots, \mathrm{e}^{\mathrm{i}\, g_n} z_n\right).$$

$\{\tau_{\mathrm{rot}}(g)\colon\ g \in G\}$ is the quasi-elliptic group.

Unitary representation of G in $\mathcal{A}^2$:

$$(\rho_{\mathrm{rot}}(g) f)(z) := f\left(\mathrm{e}^{-\mathrm{i}\, g_1} z_1, \ldots, \mathrm{e}^{-\mathrm{i}\, g_n} z_n\right).$$

Passing to the polar coordinates

Let Y be the base of $\mathbb{B}_n$ considered as a Reinhard domain:

$$Y = \left\{ v \in [0, +\infty)^n \colon \quad v_1^2 + \cdots + v_n^2 < 1 \right\}.$$

We consider Y with the Lebesgue measure μ_n.

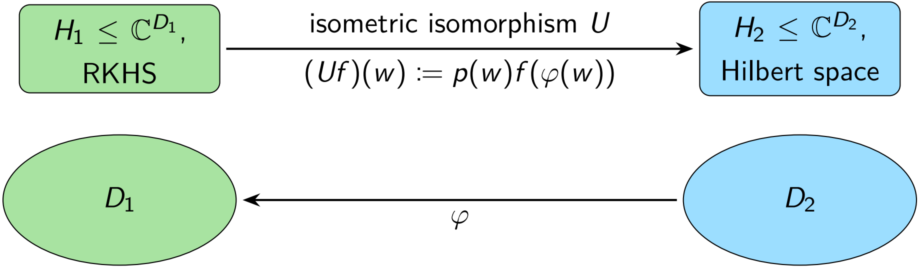
$$\varphi \colon G \times Y \to \mathbb{B}_n, \qquad \varphi(u, v) = \left(v_1 \, e^{i \, u_1}, \ldots, v_n \, e^{i \, u_n} \right).$$

For every f in $\mathcal{A}^2$, we define $Uf \in L^2(G \times Y)$,

$$(Uf)(u, v) := \sqrt{(2\pi)^n \, c_{n,\alpha} \, v_1 \cdots v_n} \, \left(1 - |v|^2\right)^{\alpha/2} \, f(\varphi(u, v)).$$

$H := U(\mathcal{A}^2)$. $\quad U \colon \mathcal{A}^2 \to H$ is an isometric isomophism.

Transformation of RK by a weighted change of variable



Then the following function is the RK of H_2:

$$K_z^{H_2}(w) = \overline{p(z)}\, p(w)\, K_{\varphi(z)}^{H_1}(\varphi(w)).$$

Passing to the polar coordinates

The reproducing kernel of H is

$$K_{x,y}(u,v) = \frac{(2\pi)^n \, c_{n,\alpha} \, (1-|y|^2)^{\alpha/2} \, (1-|v|^2)^{\alpha/2} \, \prod_{k=1}^n \sqrt{y_k v_k}}{\left(1 - \sum_{k=1}^n y_k \, v_k \, \mathrm{e}^{\,\mathrm{i}\,(u_k - x_k)}\right)^{n+\alpha+1}}.$$

We see that $K_{x,y}(u,v) = K_{0,y}(u-x,v)$.

Furthermore, U intertwines ρ_{rot} with horizontal translations:

$$\forall g \in G \qquad U \, \rho_{\mathrm{rot}}(g) = \rho_H(g) \, U.$$

Computation of L

$K_{0,y}(\cdot\,, v)$ decomposes into the Fourier series:

$$K_{0,y}(u,v) = (2\pi)^n \, c_{n,\alpha} \, (1-|v|^2)^{\alpha/2} \, (1-|y|^2)^{\alpha/2} \prod_{k=1}^{n} \sqrt{v_k y_k} \times$$

$$\times \sum_{\xi \in \mathbb{N}_0^n} \frac{\Gamma(n+|\xi|+\alpha+1)}{\xi! \, \Gamma(n+\alpha+1)} \, y^\xi \, v^\xi \, \mathrm{e}^{\mathrm{i}\,\langle u,\xi\rangle} \, .$$

For ξ in $\mathbb{N}_0^n$, the ξth Fourier coefficient is $L_{\xi,y}(v) = \overline{q_\xi(y)}\, q_\xi(v)$, where

$$q_\xi(v) = \sqrt{\frac{2^n \, \Gamma(n+|\xi|+\alpha+1)}{\xi! \, \Gamma(\alpha+1)}} \, (1-|v|^2)^{\alpha/2} \prod_{k=1}^{n} v_k^{\xi_k+\frac{1}{2}} .$$

Conclusions

In this example:

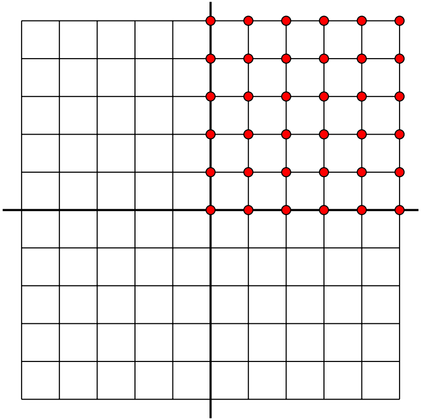
$$\widehat{G} = \mathbb{Z}^n,$$

$$\Omega = \mathbb{N}_0^n,$$

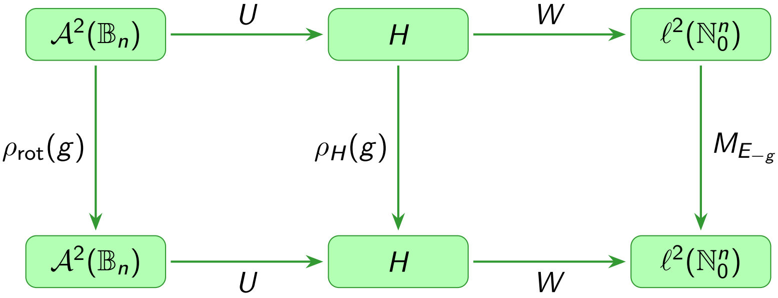
$$d_\xi = 1 \text{ for } \xi \text{ in } \mathbb{N}_0^n,$$

$$\mathcal{C}(\rho_{\text{rot}}) \cong \mathcal{C}(\rho_H) = \mathcal{V} \cong \ell^\infty(\mathbb{N}_0^n),$$

$\mathcal{V}$ is commutative.

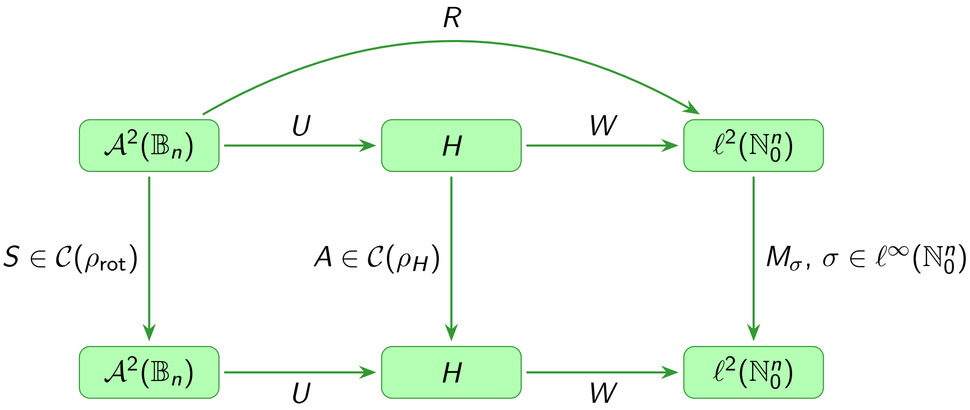


Rotations in $\mathcal{A}^2$, horizontal translations in H, and modulations in $\ell^2(\Omega)$



$$E_g(\xi) := \mathrm{e}^{\mathrm{i}\,\langle g,\xi\rangle} = \mathrm{e}^{\mathrm{i}\,(g_1\xi_1+\cdots+g_n\xi_n)}.$$

Diagonalization of the separately radial operators



The eigenvalues of separately radial Toeplitz operators in $\mathcal{A}^2(\mathbb{B}_n)$

We suppose that $a \in L^\infty(Y)$.

$$b(z_1, \dots, z_n) := a(|z_1|, \dots, |z_n|).$$

Eigenvalues' sequences ($\xi \in \mathbb{N}_0^n$):

$$\gamma_a(\xi) = \int_Y a(v)\, |q_\xi(v)|^2 \,\mathrm{d}\mu_n(v)$$

$$= \frac{\Gamma(n + |\xi| + \alpha + 1)}{\xi!\, \Gamma(\alpha + 1)} \int_{t_1 + \dots + t_n < 1} a(\sqrt{t})\, (1 - |t|_1)^\alpha\, t^\xi \,\mathrm{d}\mu_n(t).$$

This formula coincides with the formula found by Quiroga-Barranco and Vasilevski.

Outline

Bergman space on the Siegel domain

$$D_n := \left\{ w \in \mathbb{C}^n : \ \operatorname{Im}(w_n) > |w'|^2 \right\}.$$

D_n is considered with the measure

$$\frac{\Gamma(n+\alpha+1)}{4\pi^n \Gamma(\alpha+1)} \left(\operatorname{Im}(w_n) - |w'|^2\right)^\alpha \, \mathrm{d}\mu_{2n}(w).$$

$\mathcal{A}^2 :=$ the Bergman space on D_n with this measure. The reproducing kernel of $\mathcal{A}^2$:

$$K_z^{\mathcal{A}^2}(w) = \frac{1}{\left(\frac{w_n - \overline{z}_n}{2\,\mathrm{i}} - \langle w', z' \rangle \right)^{n+\alpha+1}}.$$

Quasi-parabolic subgroup of $\text{Möb}(D_n)$

$$G \;:=\; \mathbb{R}_{2\pi}^{n-1} \times \mathbb{R} \;\cong\; \mathbb{T}^{n-1} \times \mathbb{R}.$$

$\tau_{\text{qpar}}\colon\; G \to \text{Möb}(D_n)$,

$$\tau_{\text{qpar}}(g)(w) := (\mathrm{e}^{\mathrm{i}\,g_1} w_1,\; \ldots,\; \mathrm{e}^{\mathrm{i}\,g_{n-1}} w_{n-1},\; w_n + g_n) \qquad (g \in G,\;\; w \in D_n).$$

(rotations in the first $n-1$ coordinates, horizontal translation in the last coordinate)

$\tau_{\text{qpar}}(G) = \{\tau_{\text{qpar}}(g)\colon\;\; g \in G\}$ is the quasi-parabolic subgroup of $\text{Möb}(D_n)$.

Quasi-parabolic unitary representation of G in $\mathcal{A}^2(D_n)$

$G := \mathbb{R}_{2\pi}^{n-1} \times \mathbb{R}.$

$\rho_{\text{qpar}}(g)\colon \mathcal{A}^2(D_n) \to \mathcal{A}^2(D_n)$:

$$\begin{aligned}\big(\rho_{\text{qpar}}(g)f\big)(w) &:= (f \circ \tau_{\text{qpar}}(-g))(w) \\ &= f\big(\mathrm{e}^{-\mathrm{i}\,g_1}\, w_1,\ \ldots,\ \mathrm{e}^{-\mathrm{i}\,g_{n-1}}\, w_{n-1},\ w_n - g_n\big).\end{aligned}$$

Change of variables

$$G = \mathbb{R}_{2\pi}^{n-1} \times \mathbb{R}, \qquad Y = \mathbb{R}_{+}^{n}.$$

$$U\colon \mathcal{A}^2(D_n) \to H < L^2(G \times Y),$$

$$(Uf)(u,v) := \sqrt{\frac{2^{n-3}\,\Gamma(n+\alpha+1)}{\pi\,\Gamma(\alpha+1)}} \left(\prod_{j=1}^{n-1} v_j^{1/2}\right) v_n^{\alpha/2}$$

$$\times f\Big(v_1\,\mathrm{e}^{\mathrm{i}\,u_1}, \ldots, v_{n-1}\,\mathrm{e}^{\mathrm{i}\,u_{n-1}}, u_n + \mathrm{i}\,(v_n + |v'|^2)\Big).$$

$$K_{x,y}(u,v) = \frac{\mathrm{coef}_{n,\alpha}\; y_n^{\alpha/2}\; v_n^{\alpha/2}\; \prod_{j=1}^{n-1}(y_j^{1/2} v_j^{1/2})}{\left(\frac{u_n - x_n}{2\,\mathrm{i}} + \frac{v_n + y_n}{2} + \frac{|v'|^2 + |y'|^2}{2} - \sum_{j=1}^{n-1} v_j y_j\,\mathrm{e}^{\mathrm{i}\,(u_j - x_j)}\right)^{n+\alpha+1}}.$$

Fourier transform of the reproducing kernel of H

$$L_{\xi,y}(v) = \begin{cases} \overline{q_\xi(y)}\, q_\xi(v), & \xi \in \mathbb{N}_0^{n-1} \times \mathbb{R}_+, \\ 0, \text{otherwise}, \end{cases}$$

where

$$q_\xi(v) := \sqrt{\frac{2^{3n-1+2\alpha+|\xi|}\, \pi^{n+\alpha+|\xi|}\, \Gamma(n+\alpha+1)}{\Gamma(\alpha+1)\,\Gamma(n+1+\alpha+|\xi|)}} \times \left(\prod_{j=1}^{n-1} v_j^{\xi_j+\frac{1}{2}}\right) v_n^{\frac{\alpha}{2}}\, \xi_n^{\frac{n+\alpha+|\xi|}{2}}\, e^{-2\pi\xi_n(v_n+|v'|^2)}.$$

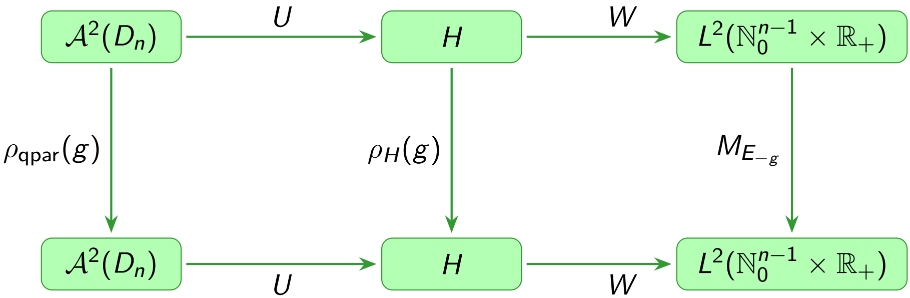
Conclusions for this case

$$\widehat{G} = \mathbb{Z}^{n-1} \times \mathbb{R}.$$

$$\Omega = \mathbb{N}_0^{n-1} \times \mathbb{R}_+.$$

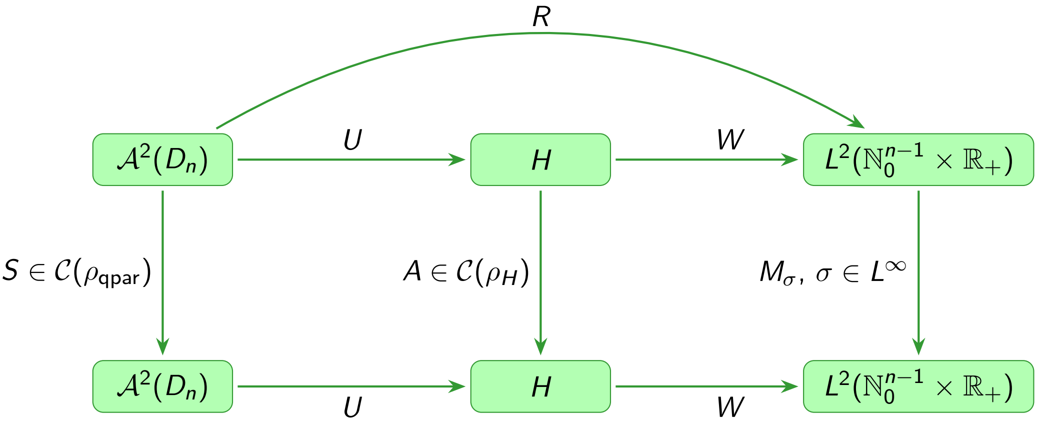
$$\mathcal{C}(\rho_{\mathrm{qpar}}) \cong \mathcal{C}(\rho_H) \cong L^\infty(\mathbb{N}_0^{n-1} \times \mathbb{R}_+).$$

Quasi-parabolic representation, horizontal translations in H, and modulations in $L^2(\Omega)$



$$E_g(\xi) := e^{i\langle g,\xi\rangle} = e^{i(g_1\xi_1+\cdots+g_{n-1}\xi_n+2\pi g_n\xi_n)}.$$

Diagonalization of ρ_{qpar}-invariant operators



Conclusions

We have applied our scheme to several classes of group-invariant operators in RKHS.

One scheme to rule them all!

If you have some RKHS, you are welcome.